

Image processing ideas for shear analysis in weak lensing

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Physics and shear

Cause:

$$\begin{bmatrix} 1 + e_1 & e_2 \\ e_2 & 1 - e_1 \end{bmatrix} \begin{pmatrix} x_{\text{Unlens}} \\ y_{\text{Unlens}} \end{pmatrix} = \begin{pmatrix} x_{\text{Observ}} \\ y_{\text{Observ}} \end{pmatrix}$$

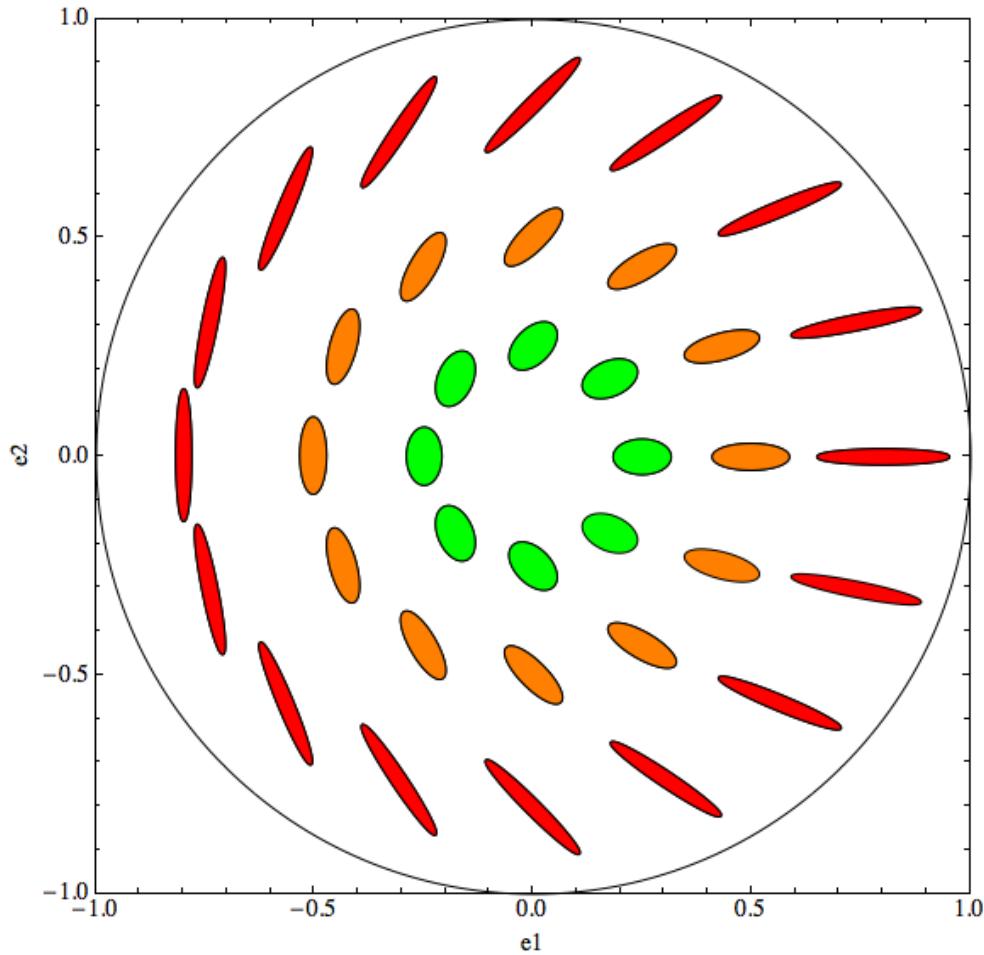
$(e_1, e_2) \leftrightarrow$ derivatives of gravitational lensing potential

Effect:

$$\begin{Bmatrix} e_1 \\ e_2 \end{Bmatrix} = \begin{Bmatrix} e \cos(2\theta) \\ e \sin(2\theta) \end{Bmatrix}$$

$$\begin{bmatrix} 1 + e_1 & e_2 \\ e_2 & 1 - e_1 \end{bmatrix} \circled{blue} = -\begin{array}{c} \text{orange ellipse} \\ \text{with semi-axes } a \text{ and } b \\ \text{and angle } \theta \end{array}$$
$$e = \frac{a - b}{a + b}$$

Life in (e_1, e_2) space



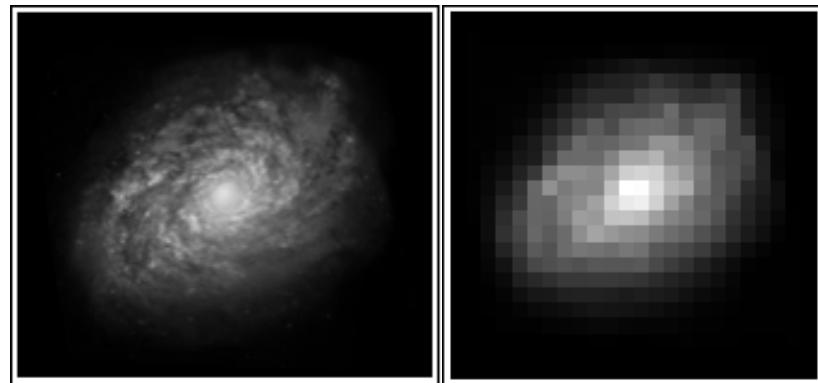
$e = 0.25, 0.5, 0.8$

Measure (e_1, e_2) of
galaxy images



Estimators of
induced
gravitational shear

Fitting > Moments



How do we go from an image to measures of (e_1, e_2) estimators?

Two basic approaches:

Moments: e.g. covariance matrix

- No assumption of detailed galaxy shape
- Bias from clipping in tails
- Bad noise performance, or use weighting (unknown, tricky)

Fitting: with parameterized form

- Works on any piece of image
- Better noise resistance; but potential noise bias
- True shapes unknown; noise effect depends on shape (Great3)

Wish list?

Can we find an image processing technique that will let us

- Push every image toward a known fit-able shape, while still *preserving* eccentricity information
- Take advantage of the properties of pixel by pixel noise (whiter shade of pale)

Convolutions/Correlations

Two interesting, suggestive mathematical facts:

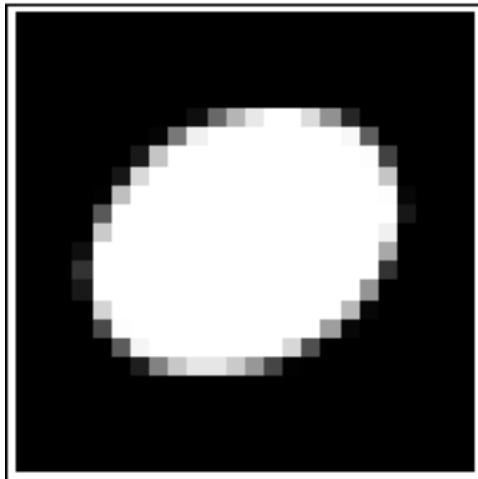
- Repeated auto-correlation/convolution converges toward a Gaussian, while preserving variance ratios
- Auto-correlation of white noise is a predictable $\delta()$ function

Four experiments:

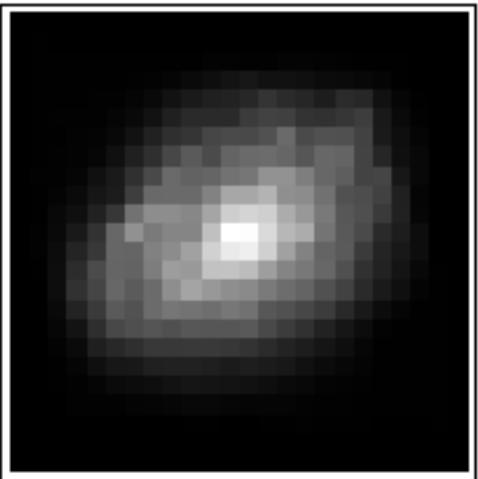
Using auto-correlation/convolution + fitting technique:

- Look at three very different images with same eccentricity, see how noise performance varies
- Look at noise behavior at high ellipticity
- Using Gaussian images, look at resistance to tail clipping
- Shear an assortment of Gaussians, look at shear recovery on average

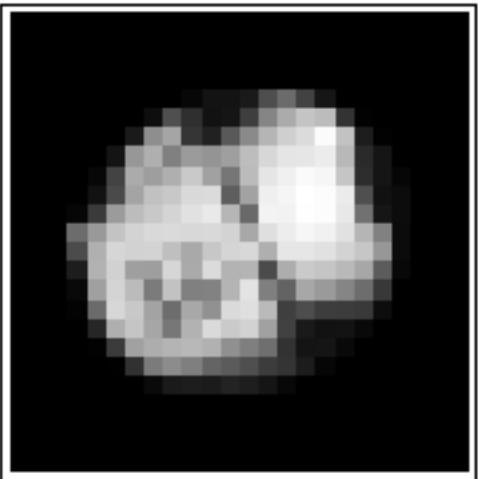
E1: Meet our contestants



DISH

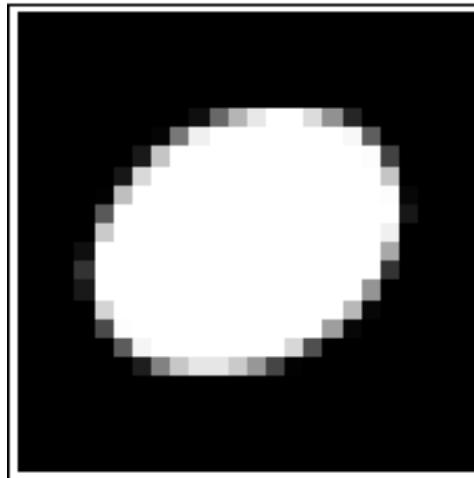


NGC4414

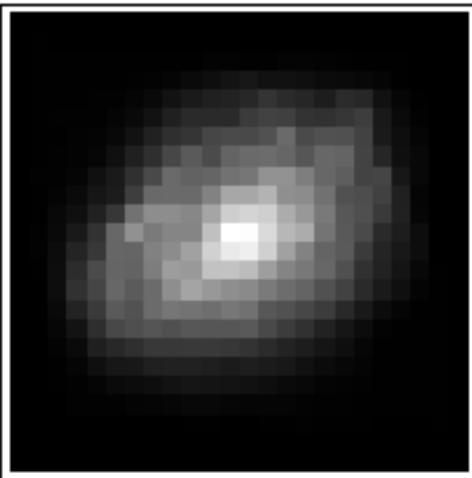


RMN1970

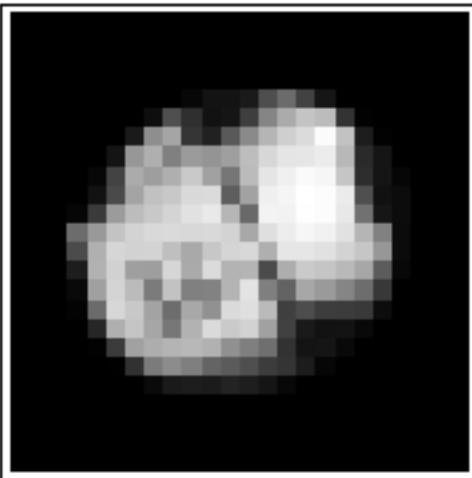
E1: Meet our contestants



DISH



NGC4414



RMN1970

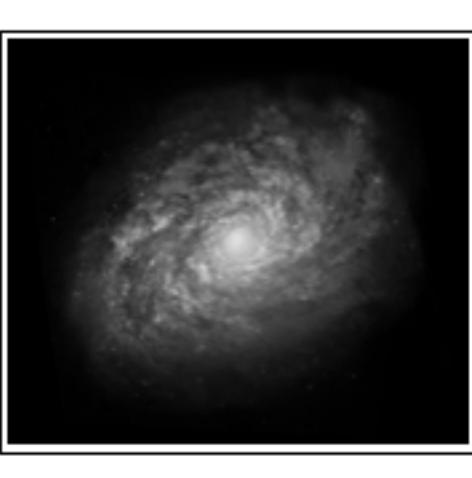
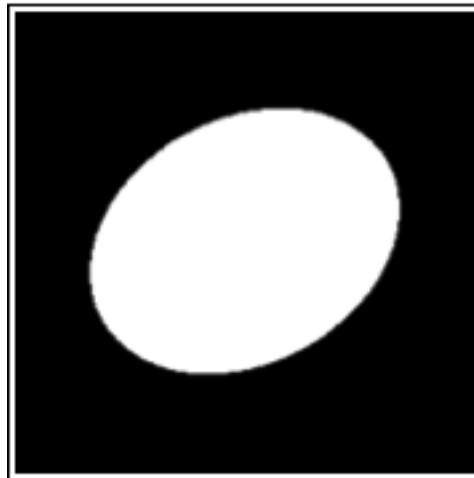
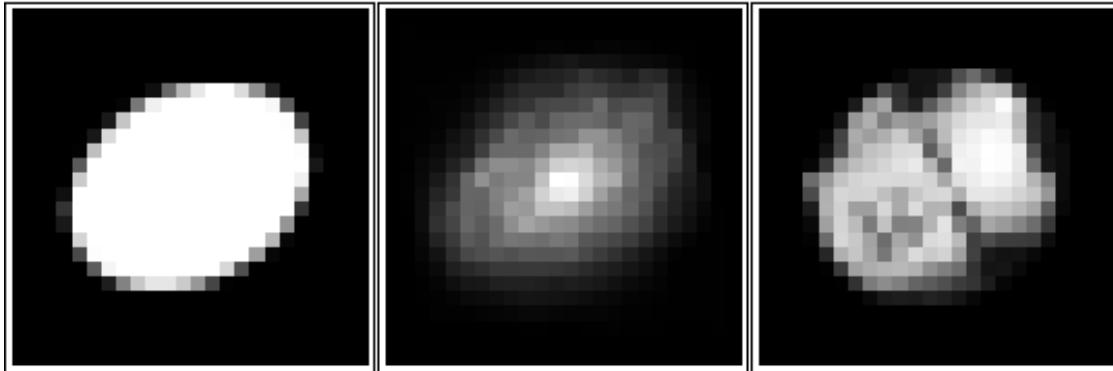


Image processing, first order

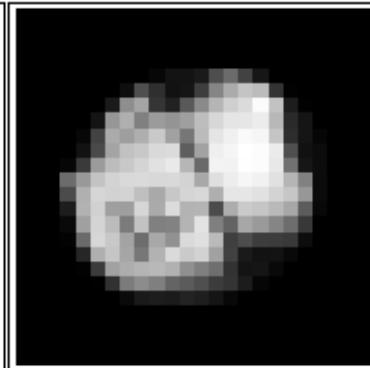
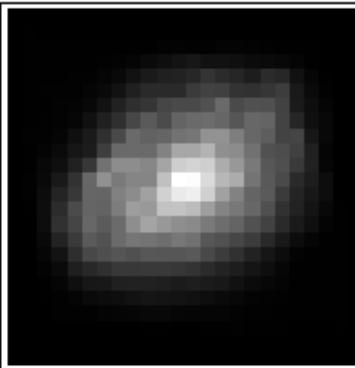
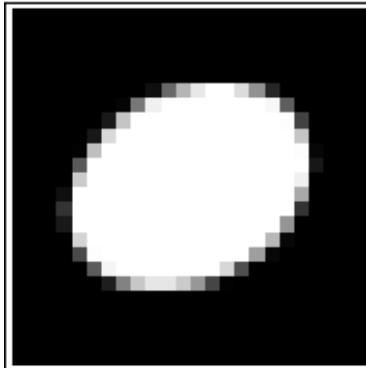


Original

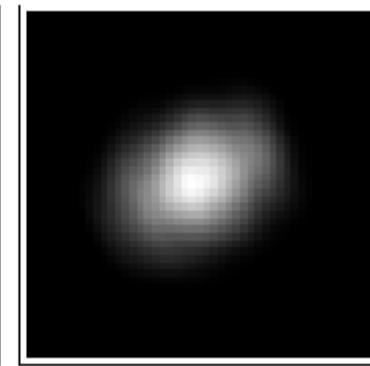
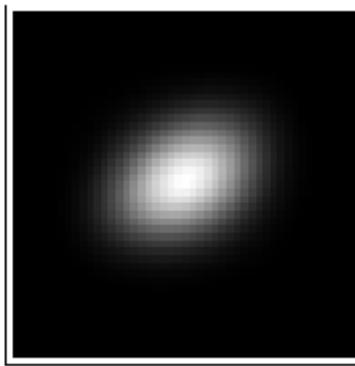
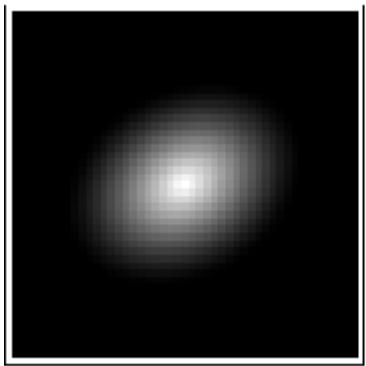
Auto - convolution

Auto - correlation

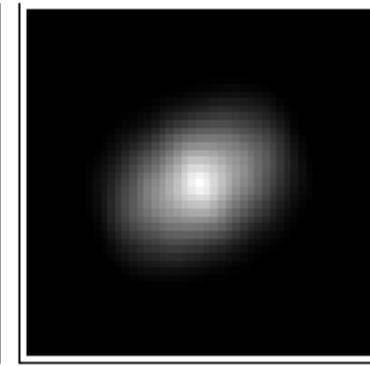
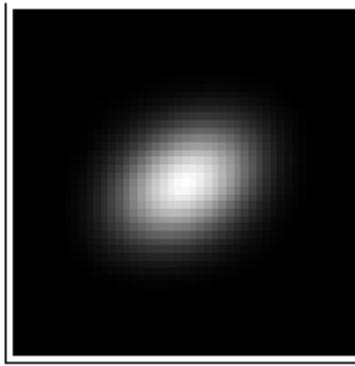
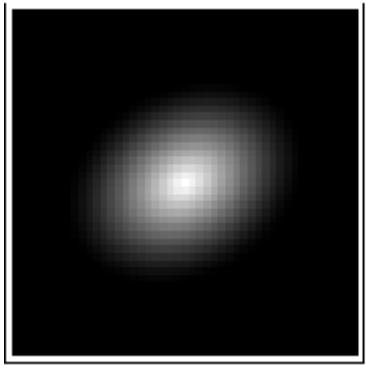
Image processing, first order



Original

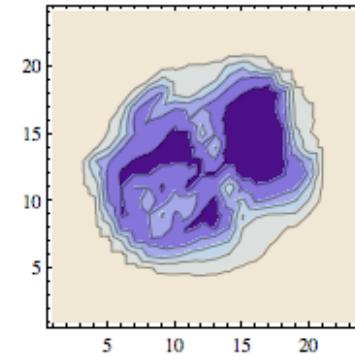
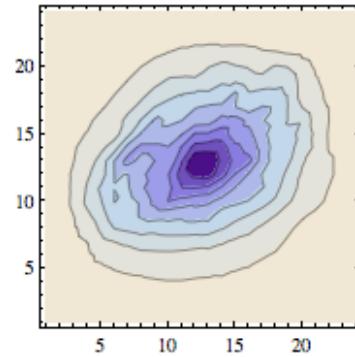
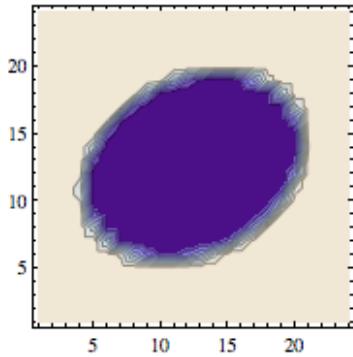


Auto - convolution

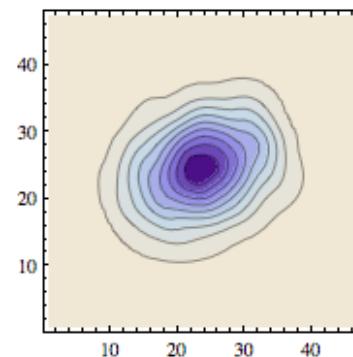
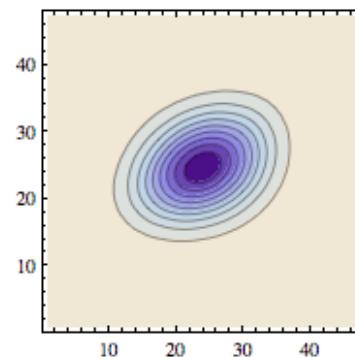
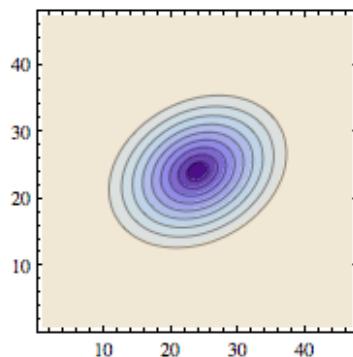


Auto - correlation

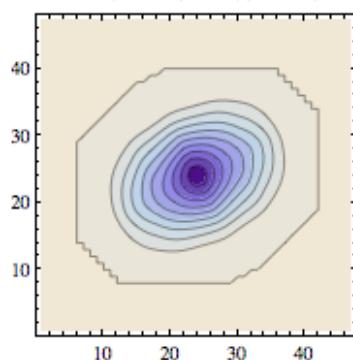
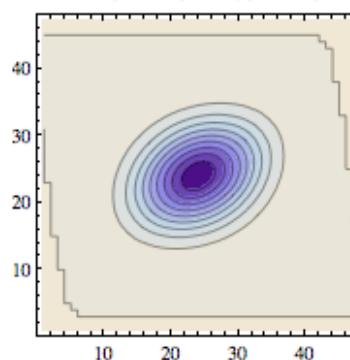
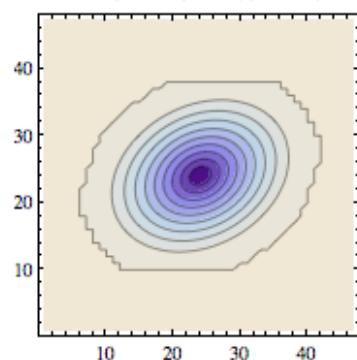
Image processing, first order



Original

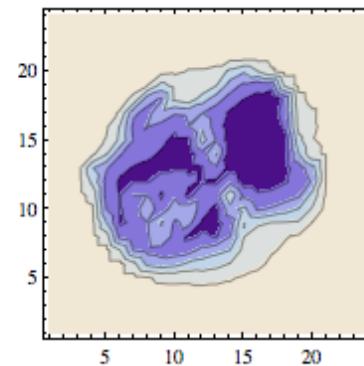
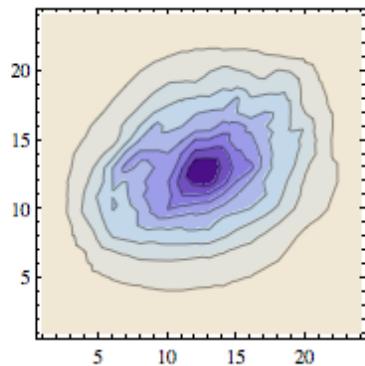
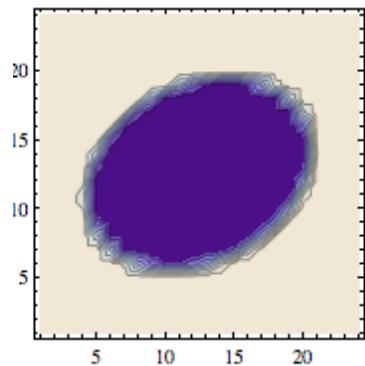


Auto - convolution

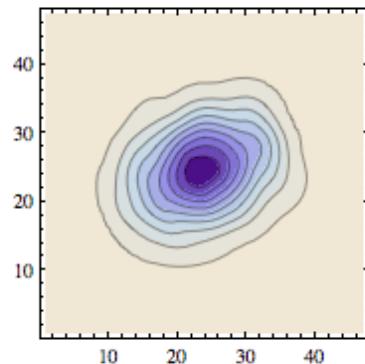
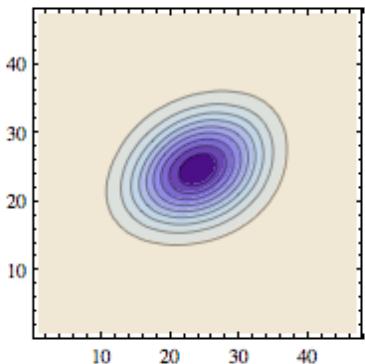
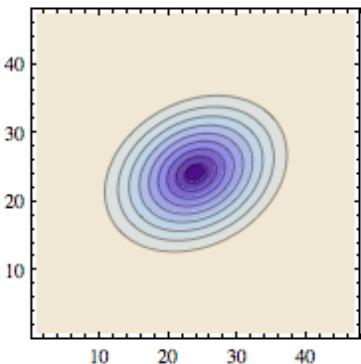


Auto - correlation

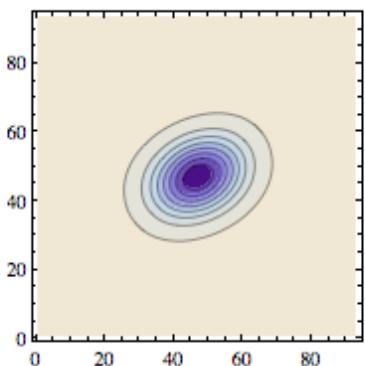
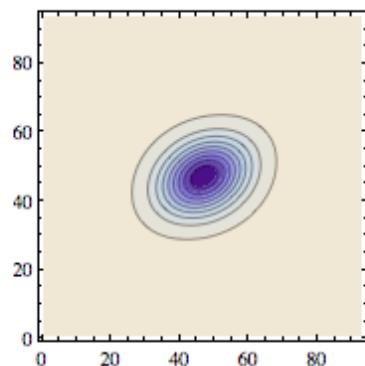
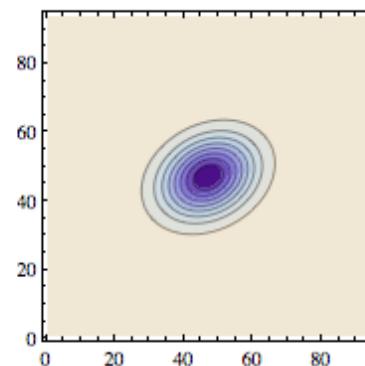
Image processing, second order



Original

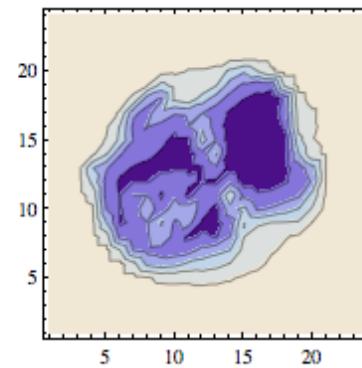
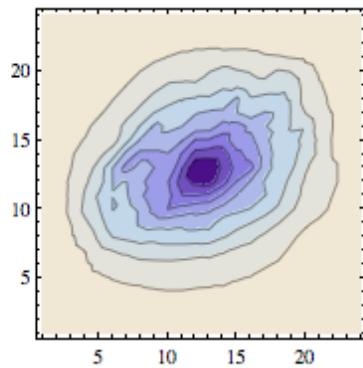
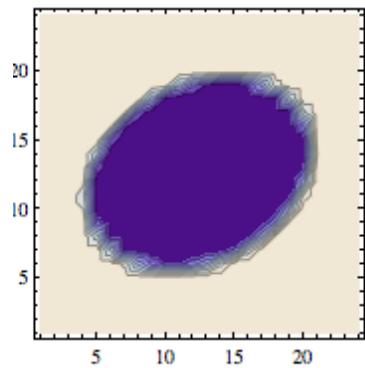


First auto -
correlation

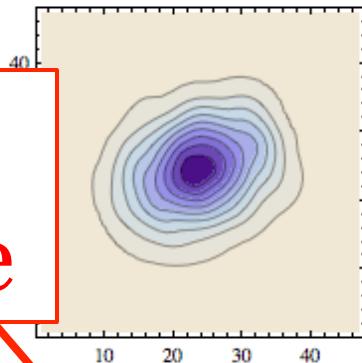
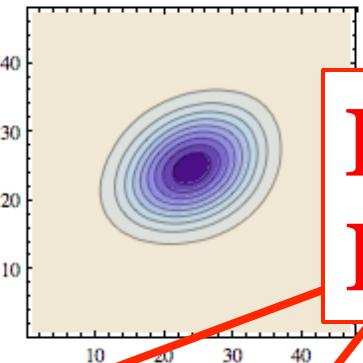
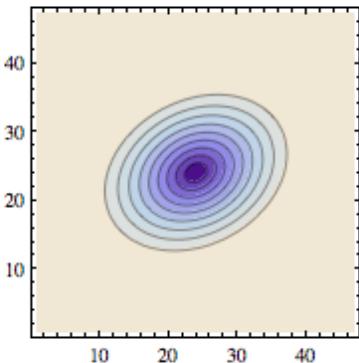


Second
auto -
correlation

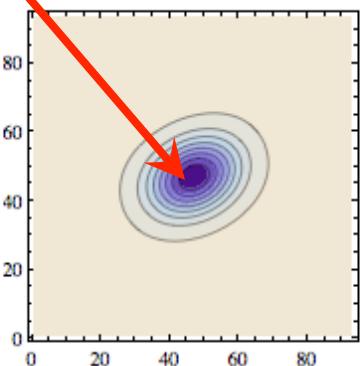
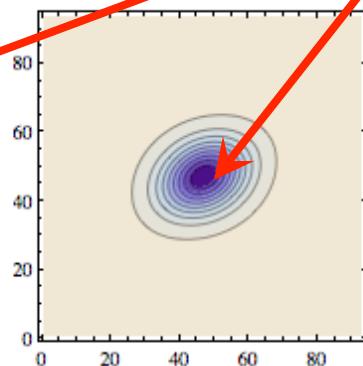
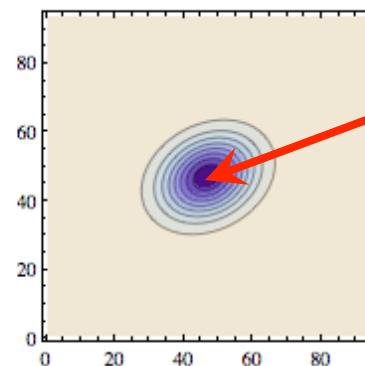
Image processing, second order



Original



Fit
Here



First auto -
correlation

Second
auto -
correlation

$$S/N \equiv \frac{\sum_{i,j} (\text{Image}_{i,j})^2}{\sum_{i,j} (\text{Noise}_{i,j})^2}$$

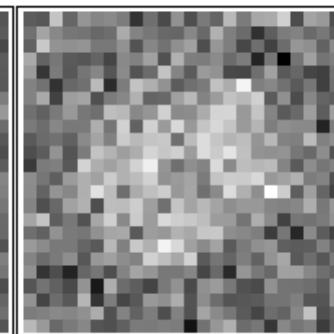
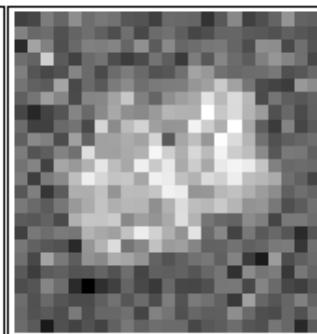
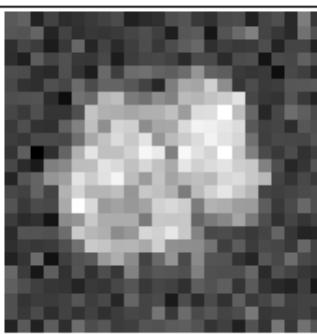
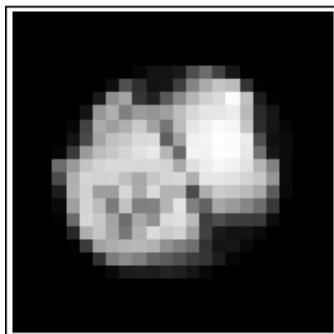
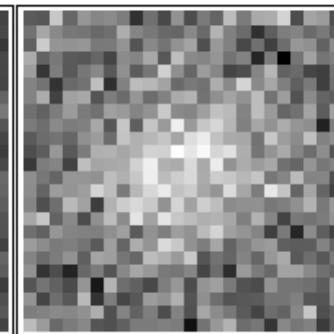
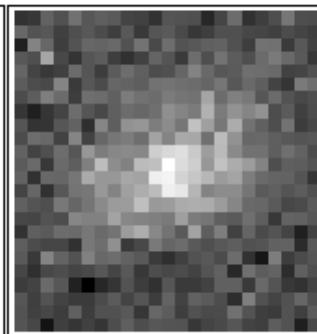
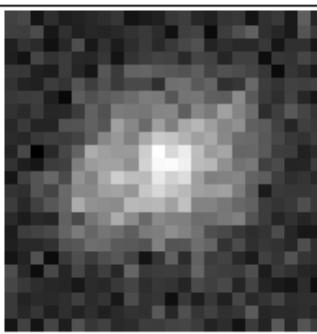
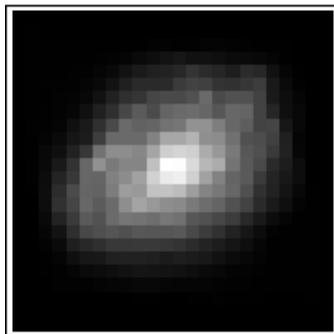
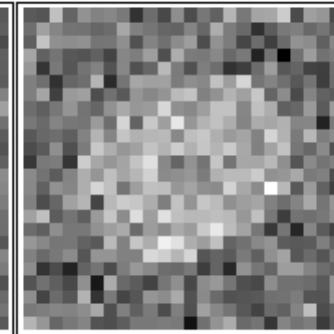
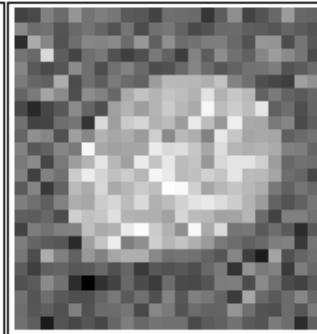
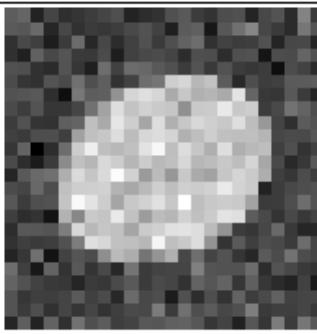
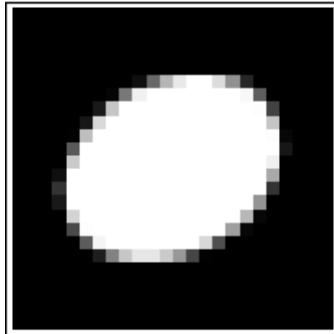
Signal/Noise

∞

10/1

3/1

1/1



$$S/N \equiv \frac{\sum_{i,j} (\text{Image}_{i,j})^2}{\sum_{i,j} (\text{Noise}_{i,j})^2}$$

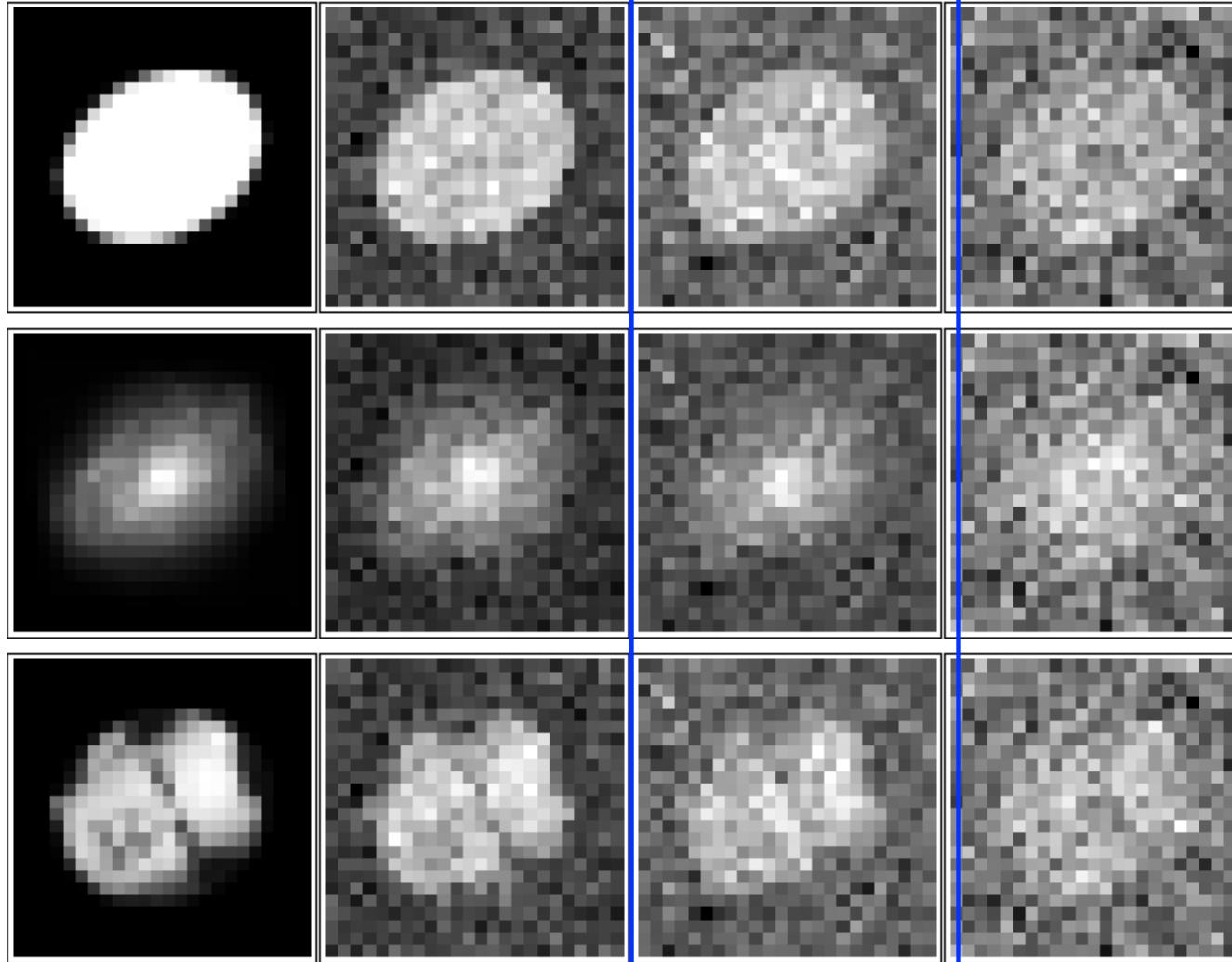
Signal/Noise

∞

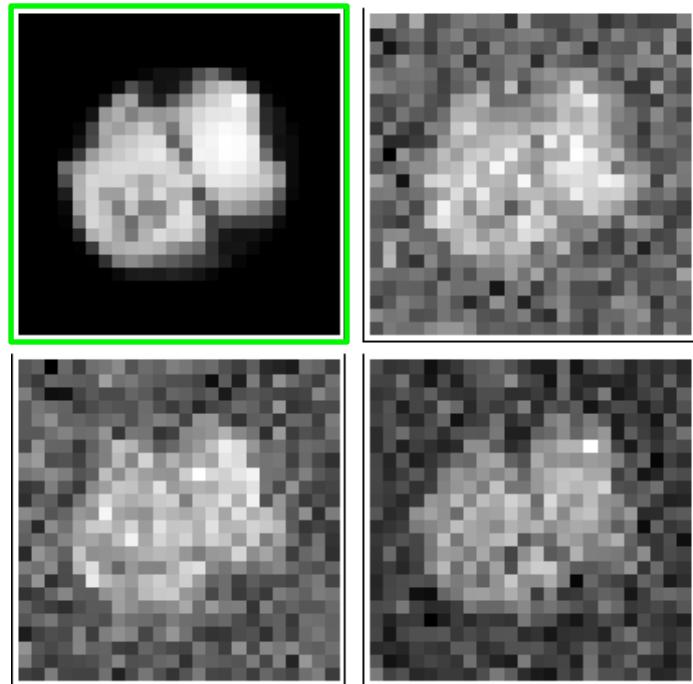
10/1

3/1

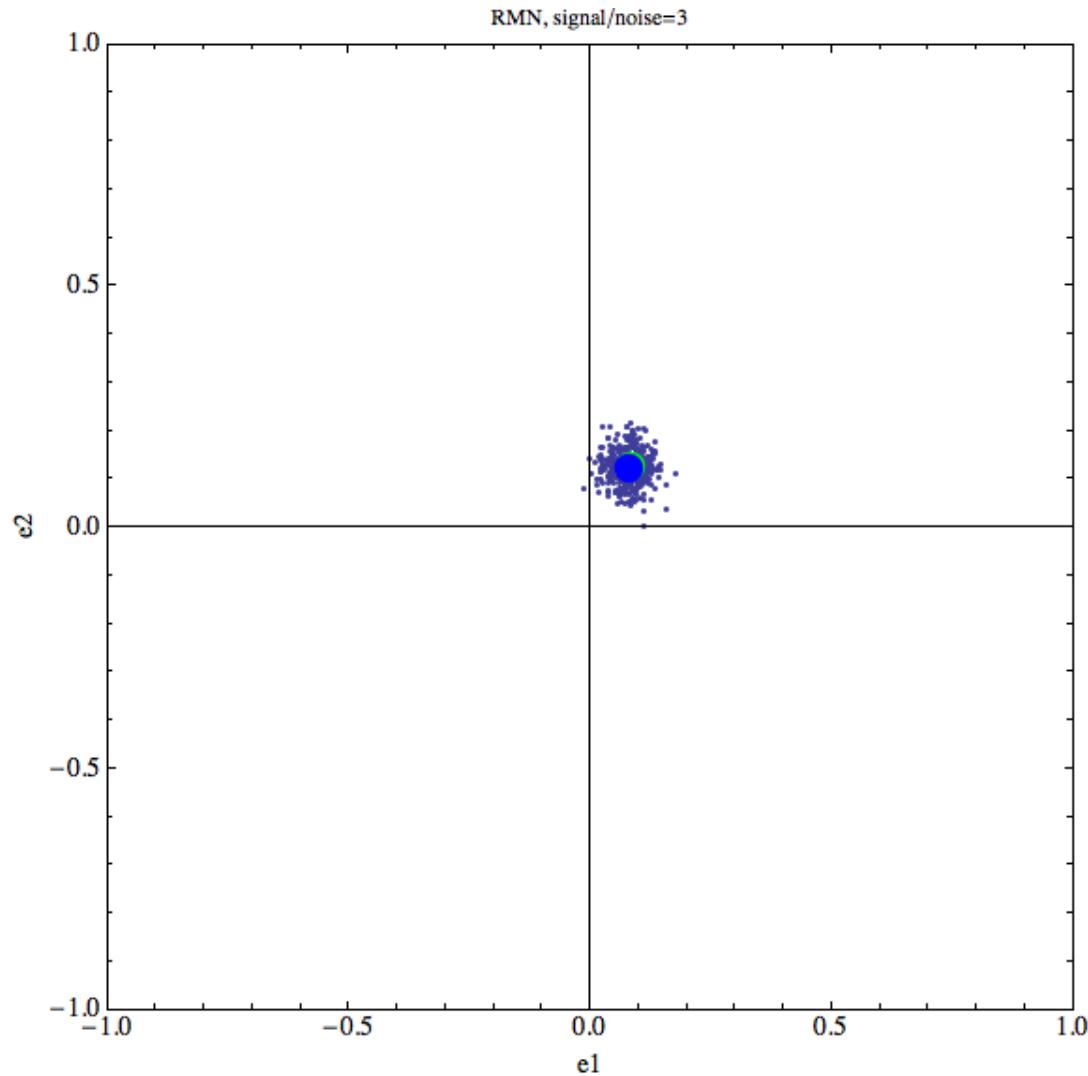
1/1



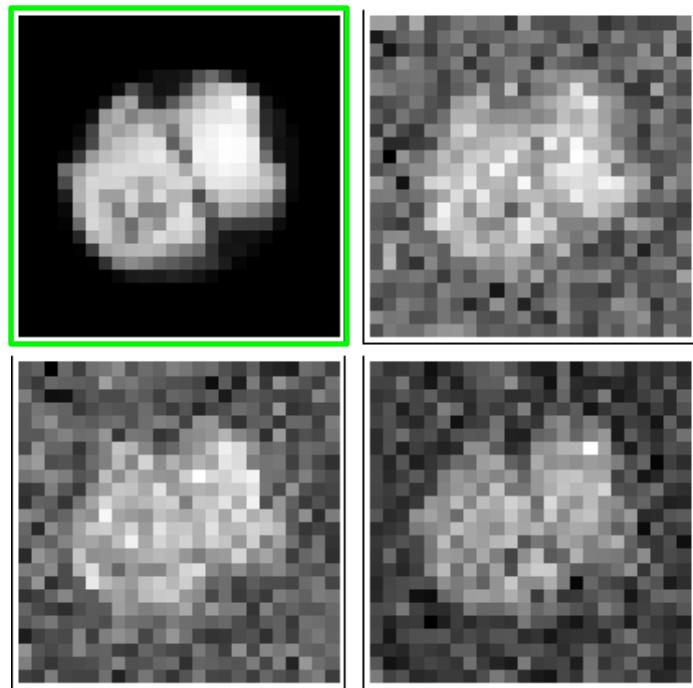
(e1,e2) using auto-conv+corr



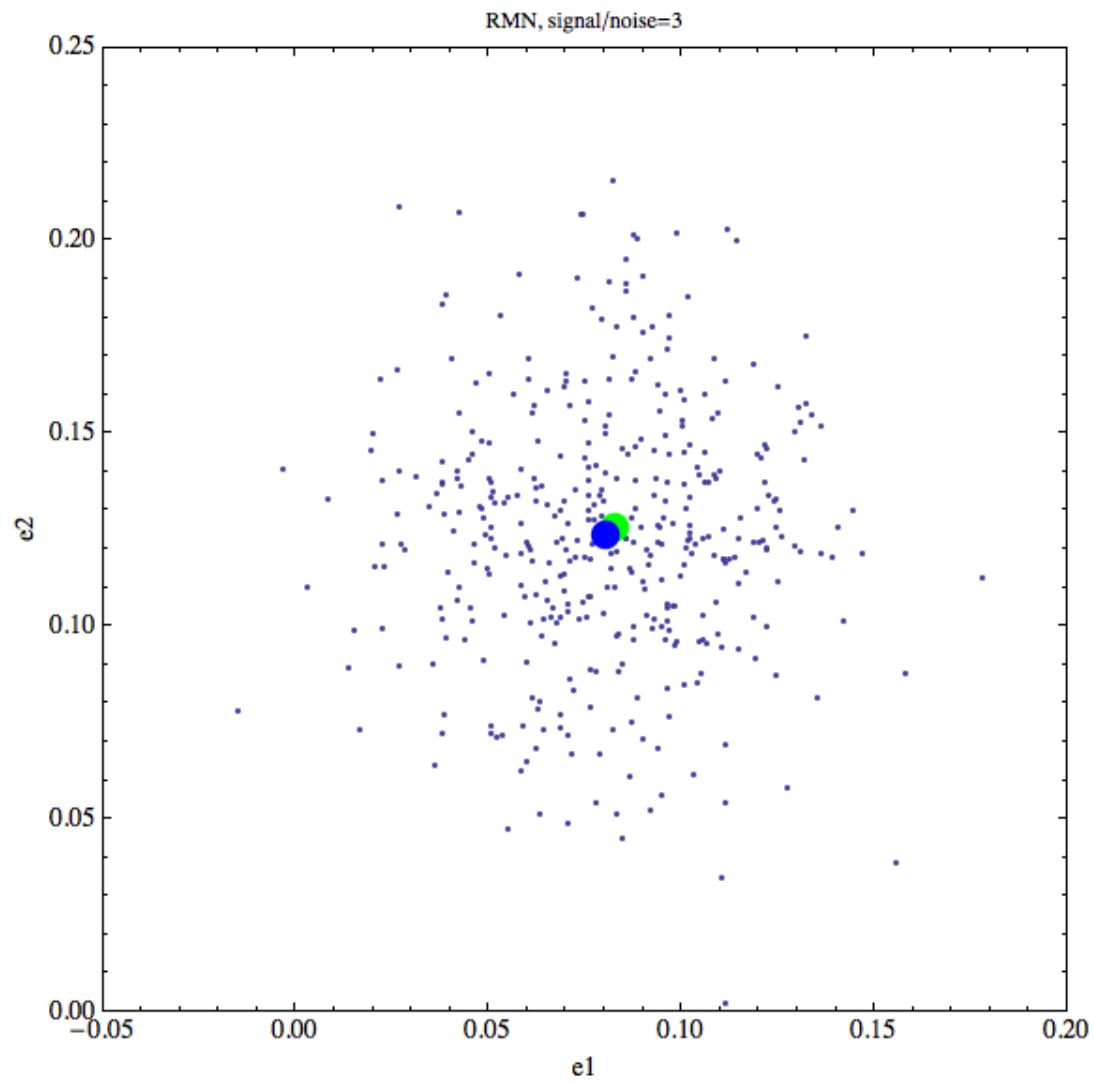
1. Take autoCorr(autoConv(Im))
2. Fit central cap with elliptical peak shape
3. Extract ellipses' (e1,e2)



(e1,e2) using auto-conv+corr

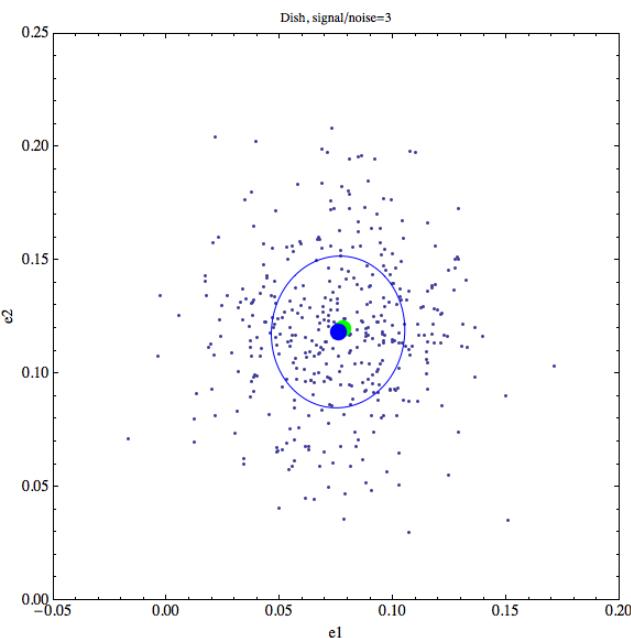


1. Take autoCorr(autoConv(Im))
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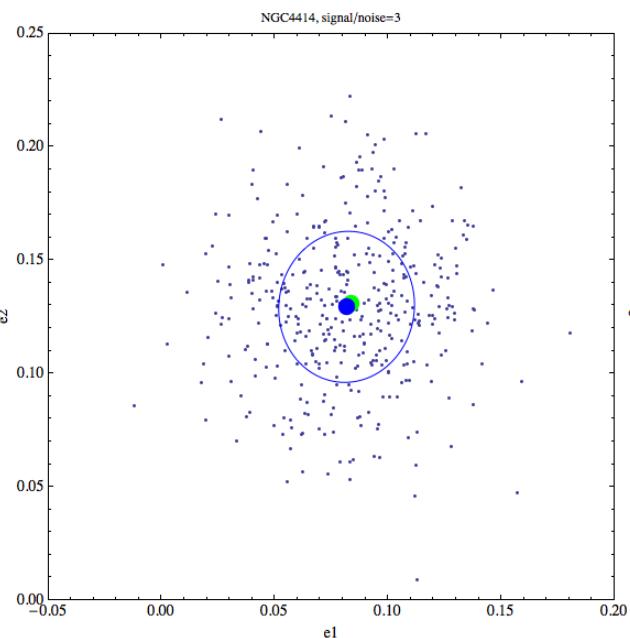


Three images' (e1,e2) spreads

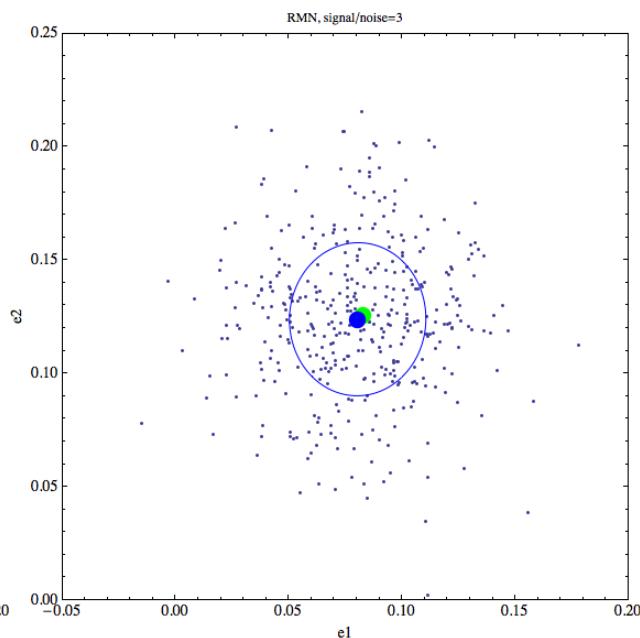
DISH



NGC4414



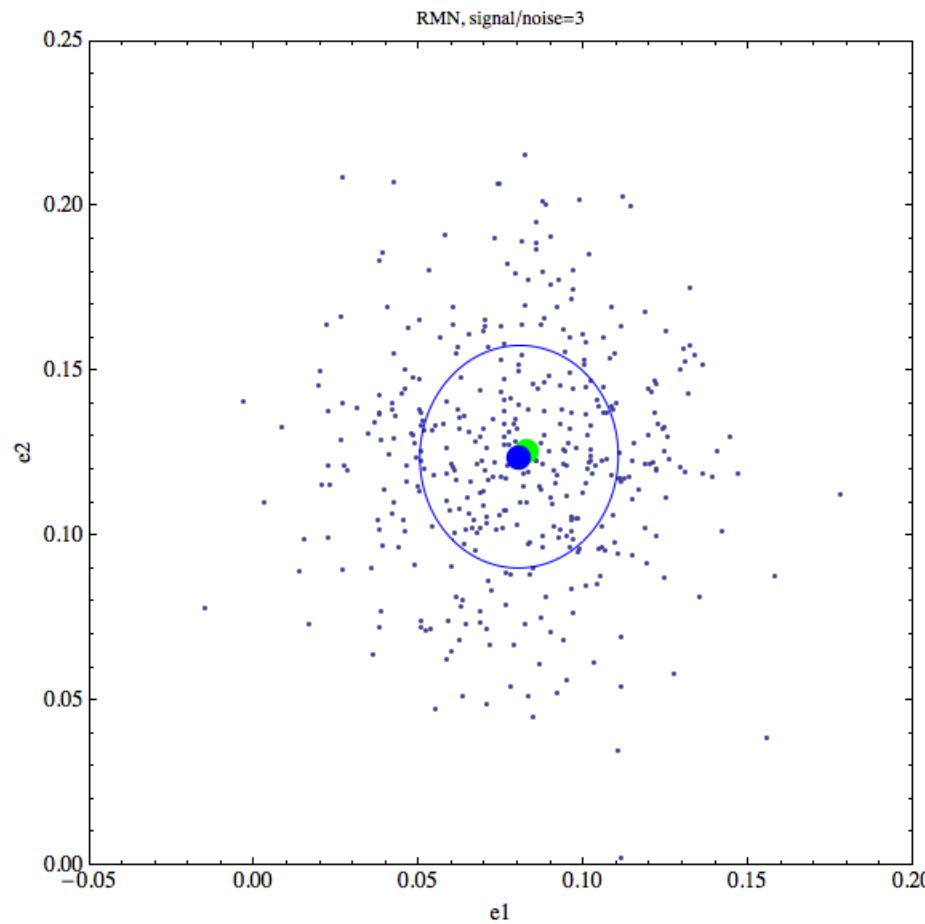
RMN1970



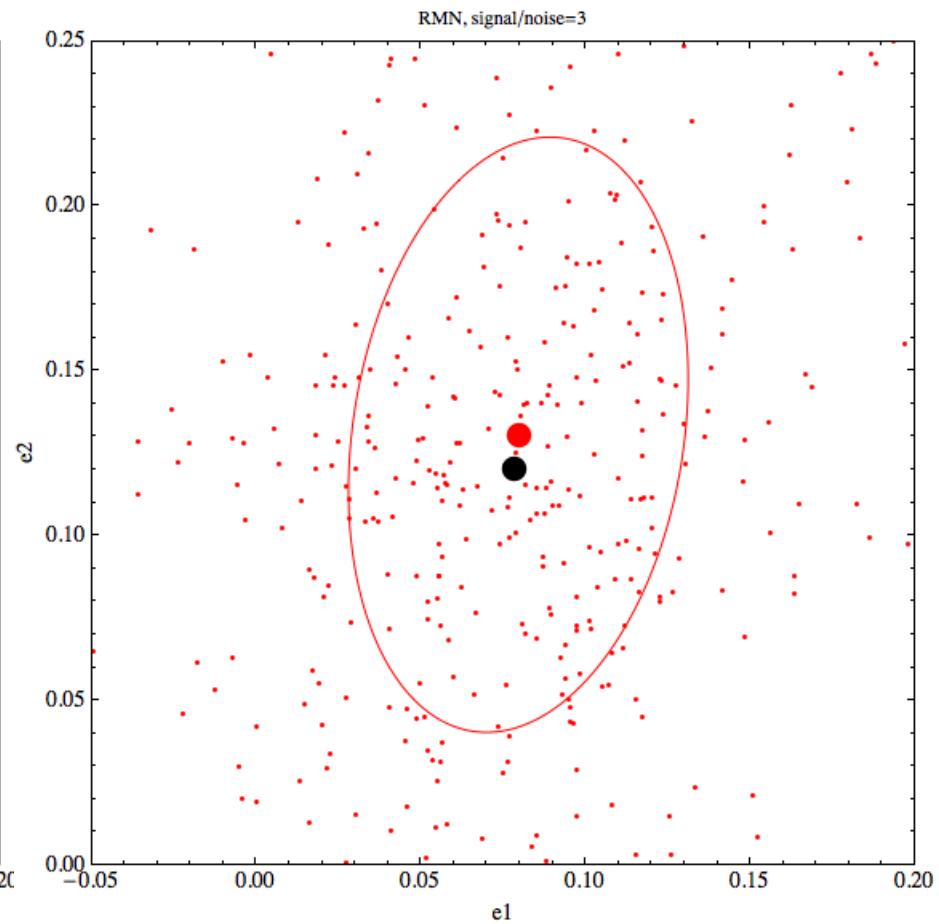
Noise spread and bias very similar! using auto-corr/conv technique, for three initially very different images.

Auto-conv/corr vs moments

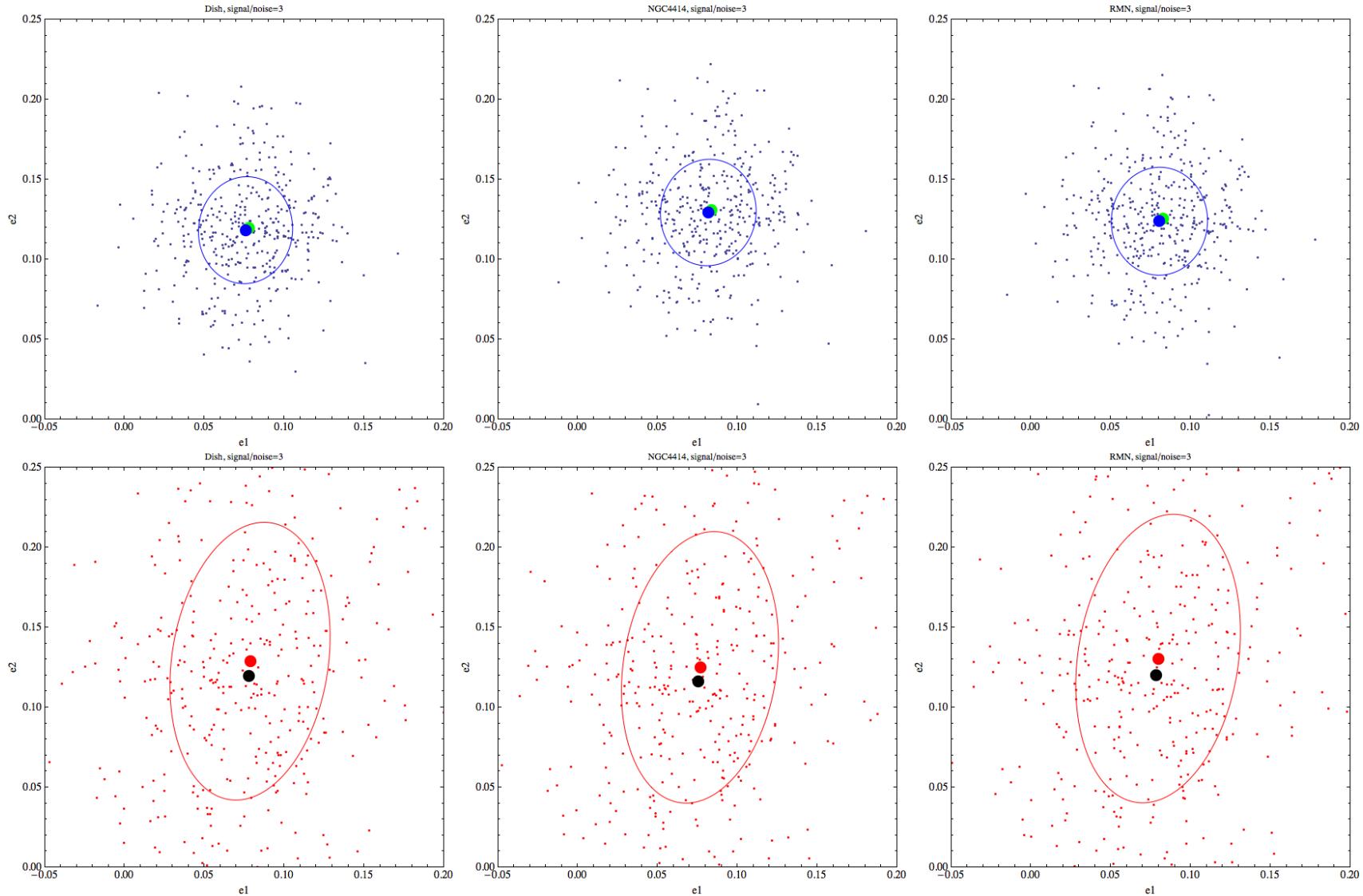
RMN1970 (e1,e2) using
2nd order conv/corr



RMN1970 (e1,e2) using
unweighted covariances



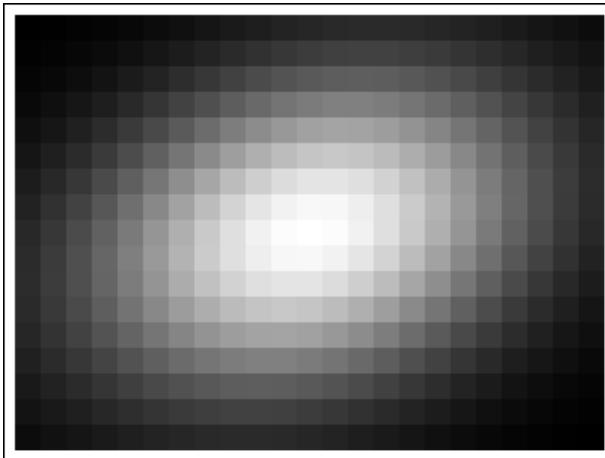
Three Full Noise Spreads



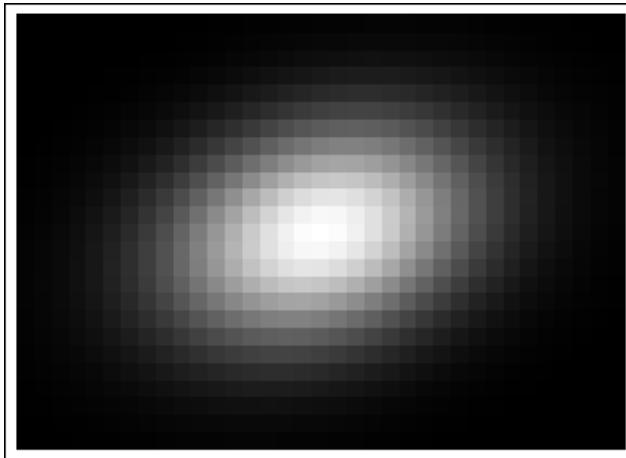
E3: Trimming Gaussians

Noise-less pure Gaussian profile images

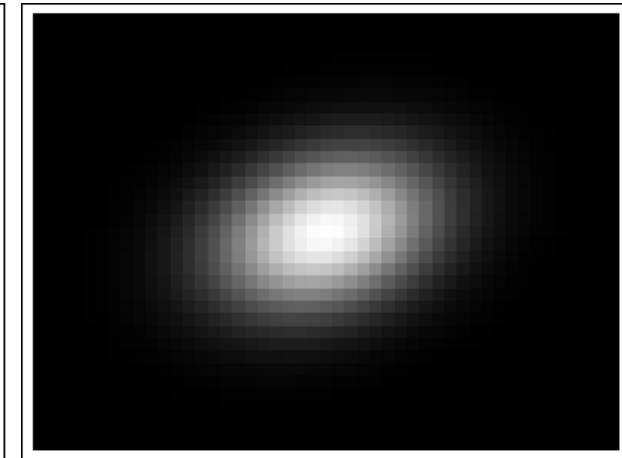
2σ window



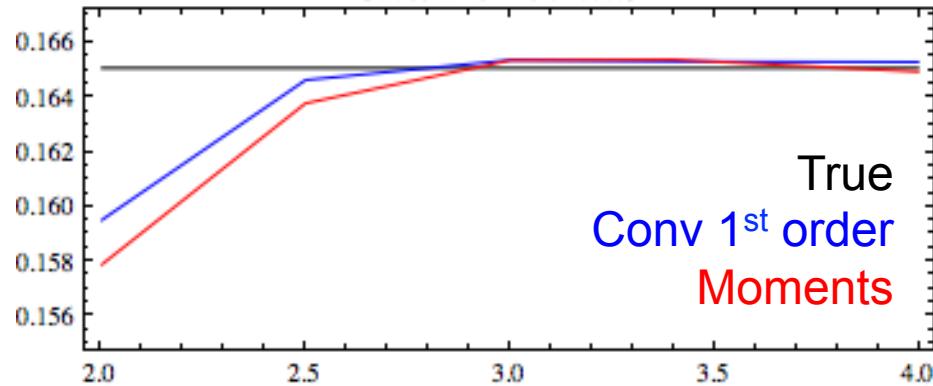
3σ window



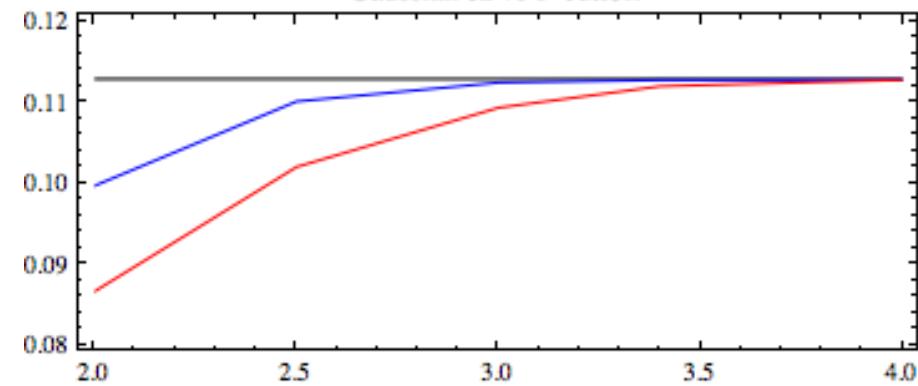
4σ window



Gaussian e1 vs σ cutoff



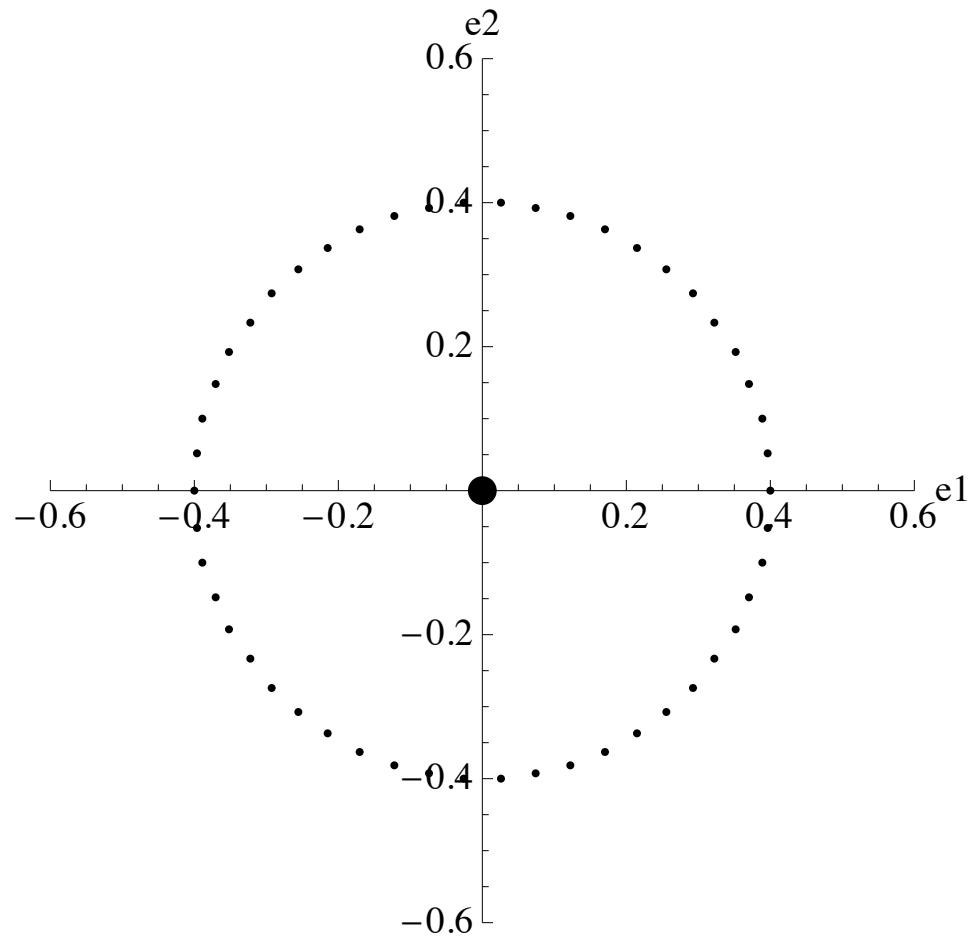
Gaussian e2 vs σ cutoff



Shear recovery

$$\begin{bmatrix} 1 + 0.15 & 0 \\ 0 & 1 - 0.15 \end{bmatrix}$$

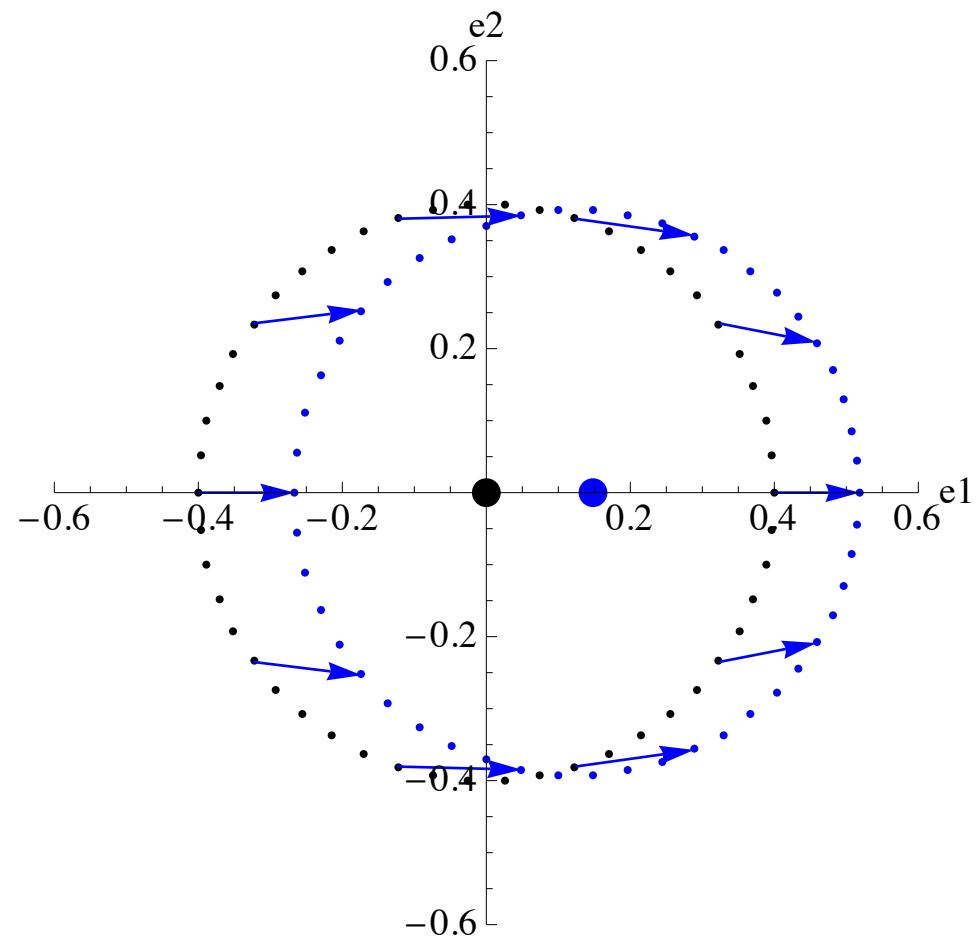
Take a collection of images with (e_1, e_2) spread uniformly around a circle, and re-shear each one by the same transformation.



Shear recovery

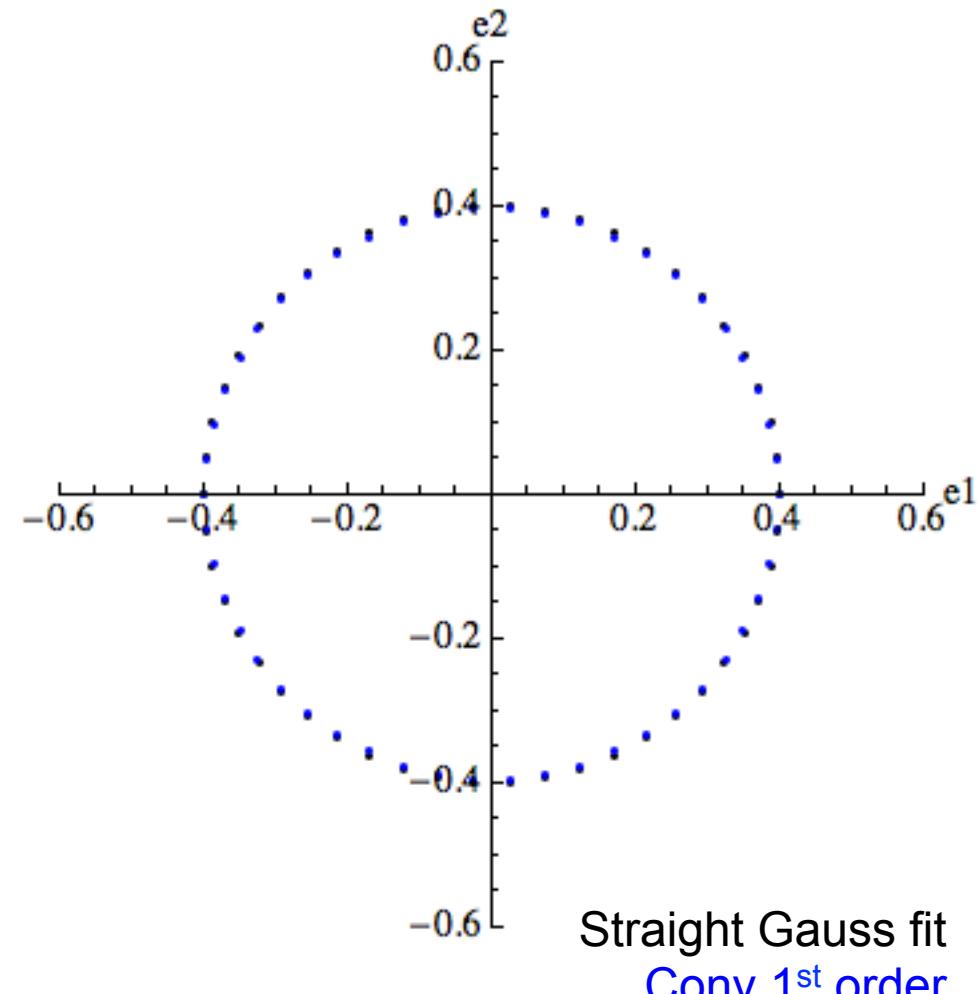
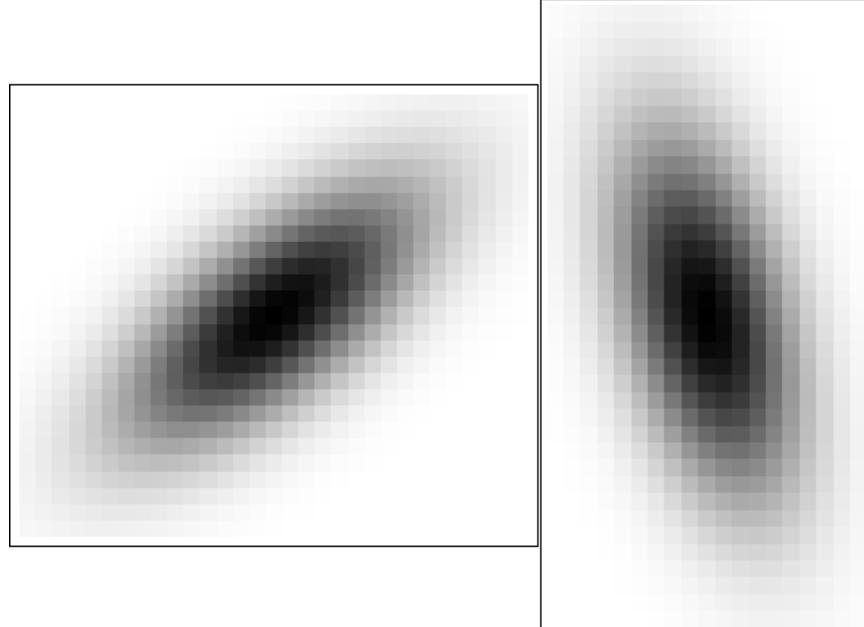
$$\begin{bmatrix} 1 + 0.15 & 0 \\ 0 & 1 - 0.15 \end{bmatrix}$$

The arithmetic average of (e_1, e_2) after shearing recovers the shear exactly, even though each image is changed differently.



Ring of Gaussians

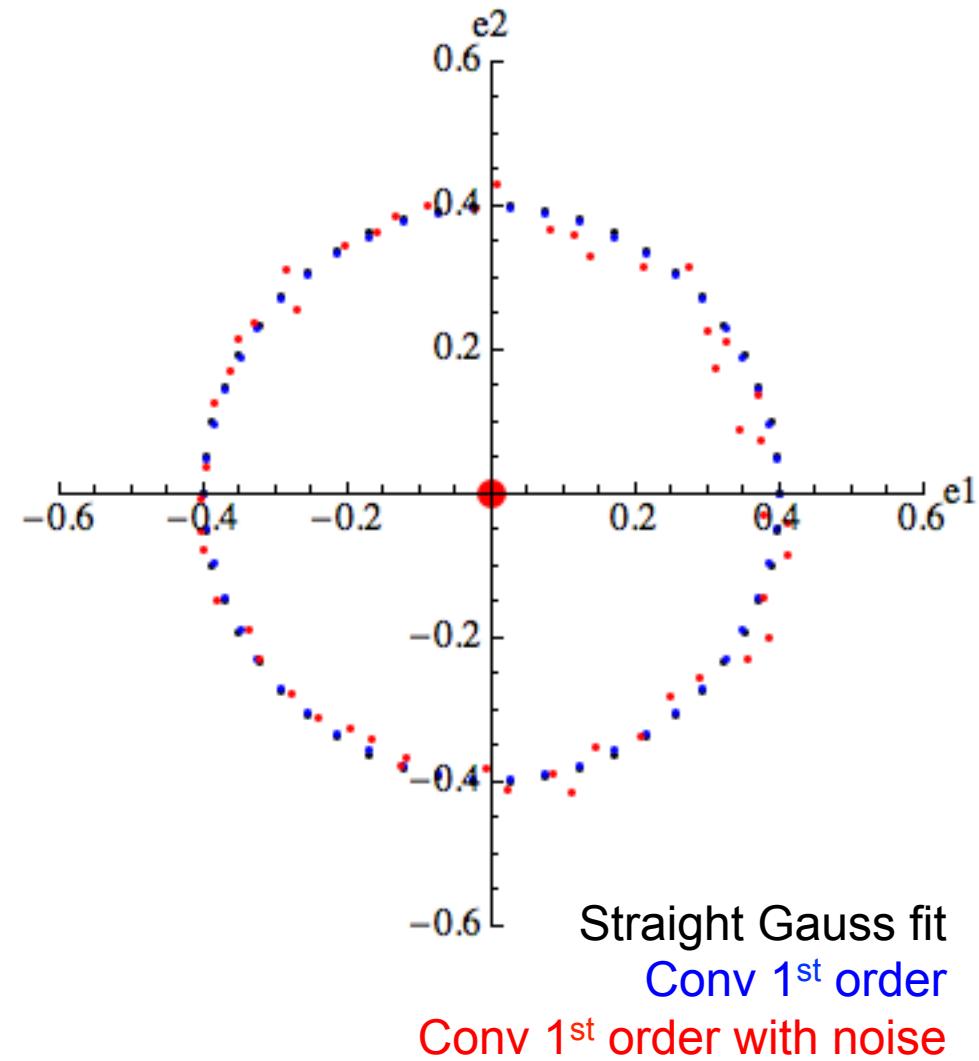
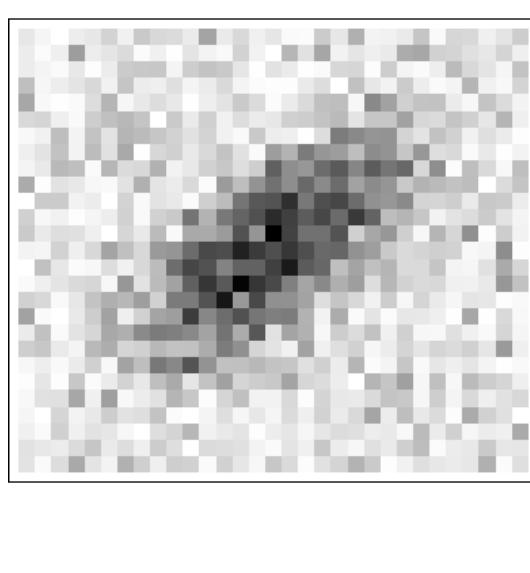
Generate a herd of 50 Gaussian profile images, clipped at 2.5σ , $e=0.4$ and rotated uniformly.



Straight Gauss fit
Conv 1st order

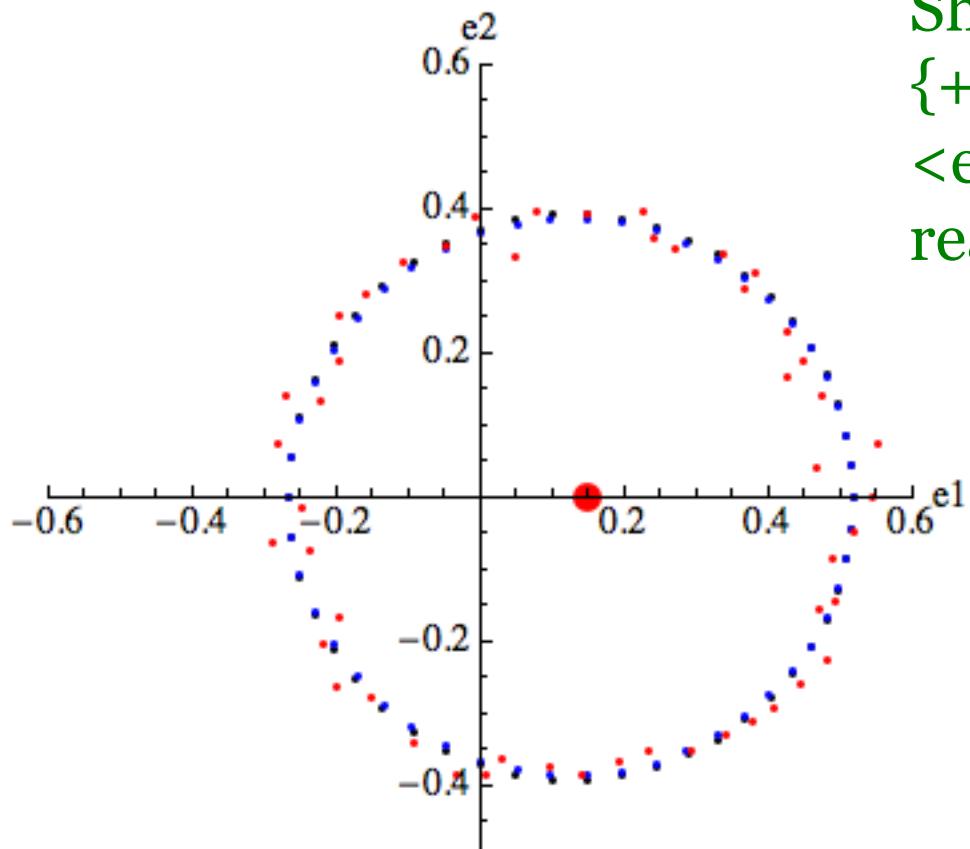
Ring of noisy Gaussians

Add noise to each of the herd at S/N=3, look at averaged $\langle e_1 \rangle$ and $\langle e_2 \rangle$



Straight Gauss fit
Conv 1st order
Conv 1st order with noise

E4: Gaussian shear recovery

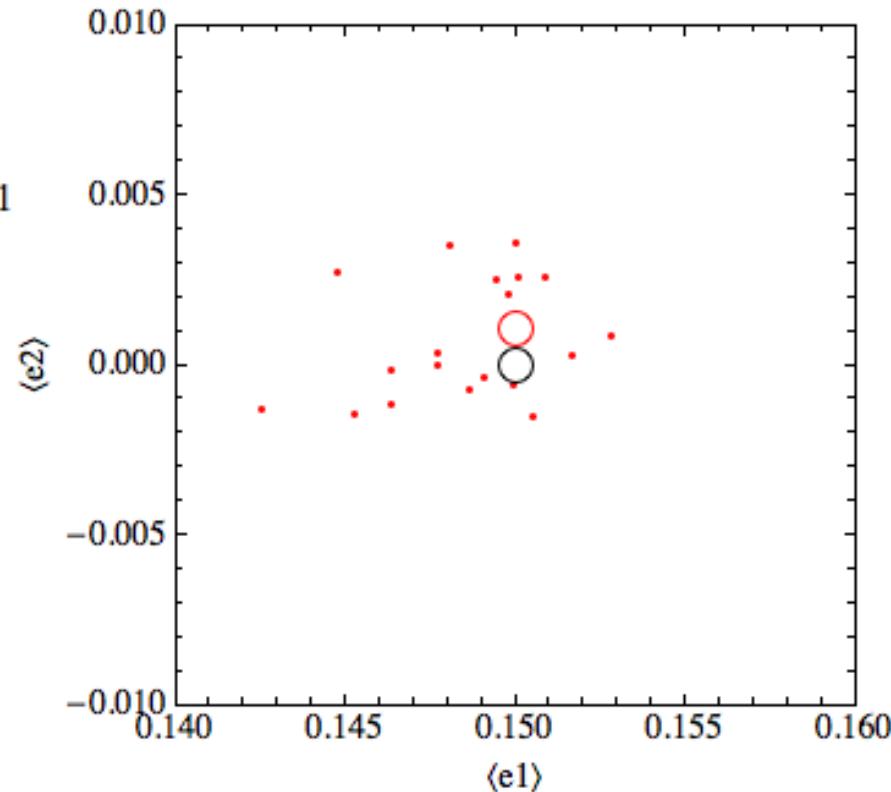


Straight Gauss fit

Conv 1st order

Conv 1st order with noise

Shear each Gaussian by $\{+0.15, 0\}$ and repeat; look at $\langle e_1 \rangle$ and $\langle e_2 \rangle$ over 20 noise realizations, not bad recovery!



Summary

- Auto-correlation/convolution image processing promises best of both worlds: fitting procedure without models, independent of original shapes
- Performance right out of the box looks surprisingly good!
- Still have to deal with effects of optics (psf) and pixelization; possible plans....